

Absolute Value Equations & Inequalities



Absolute Value

1. The **absolute value** of a number is its distance from zero on the number line.

In other words, $|-2|$ is asking "How far away from zero is the number -2?"

a. $|-2| = \underline{2}$

$|2| = \underline{2}$

b. $|\frac{-2}{3}| = \underline{\frac{2}{3}}$

$|\frac{2}{3}| = \underline{\frac{2}{3}}$

c. $|-1.5| = \underline{1.5}$

$|1.5| = \underline{1.5}$

2. Consider the expressions $|3 - 8|$ and $|3| - |8|$. Are they the same or different?

$|3 - 8| = |5| = 5$

$|3| - |8| = 3 - 8 = -5$

Different.

3. Now consider the expression $|x| = 6$. What does this expression mean?

x is a number that is 6 units from zero.

x could be 6 or -6.

Absolute Value Equations



4. Solve each linear absolute value equation. Show your work.

a. $|x + 7| = 3$

$x + 7 = 3$ or $x + 7 = -3$

$x = -4$

$x = -10$

b. $|x - 9| = 12$

$x - 9 = 12$ or $x - 9 = -12$

$\frac{x - 9}{+9 \quad +9} = 21$

$\frac{x - 9}{+9 \quad +9} = -3$

c. $|3x + 7| = -8$

~~no~~ No solution. -8 is not a distance.





8. Solve each linear absolute value equation.

$$\begin{aligned} \text{a. } |x| + 16 &= 32 \\ &\quad \underline{-16 \quad -16} \\ |x| &= 16 \end{aligned}$$

$$x = 16 \quad \text{or} \quad x = -16$$

$$\begin{aligned} \text{b. } 23 &= |x - 8| + 6 \\ &\quad \underline{-6 \quad \quad -6} \\ 17 &= |x - 8| \end{aligned}$$

$$x - 8 = 17 \quad \text{or} \quad x - 8 = -17$$

$$x = 25 \quad \text{or} \quad x = -9$$

$$\begin{aligned} \text{c. } 3|x - 2| &= 12 \\ |x - 2| &= 4 \end{aligned}$$

$$x - 2 = 4 \quad \text{or} \quad x - 2 = -4$$

$$x = 6 \quad \text{or} \quad x = -2$$

Consider isolating the absolute value part of the equation before you rewrite as two equations.



$$\begin{aligned} \text{d. } 35 &= 5|x + 6| - 10 \\ 45 &= 5|x + 6| \\ 9 &= |x + 6| \end{aligned}$$

$$\begin{aligned} x + 6 &= 9 \quad \text{or} \quad x + 6 = -9 \\ x &= 3 \quad \text{or} \quad x = -15 \end{aligned}$$

~~16~~

Absolute Value Inequalities



9. Consider the expression $|x| < 6$. What does this expression mean?

x is a number less than 6 units from 0.

$$-6 < x < 6$$

10. Graph the solution to the expression on the number line.

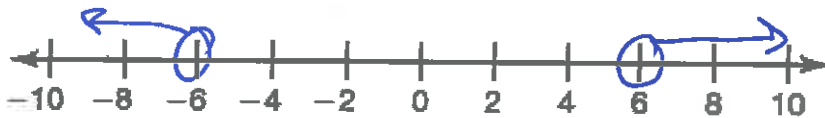


11. What does the expression $|x| > 6$ mean?

x is a number more than 6 units from zero

$$x > 6 \text{ or } x < -6$$

12. Graph the solution to the expression on the number line.



1. Solve the linear absolute value inequality by rewriting it as an equivalent compound inequality. Then graph your solution on the number line.

a. $|x + 3| < 4$



$$\begin{array}{l} -4 < x+3 < 4 \\ -3 \quad -3 \quad -3 \\ \hline -7 < x < 1 \end{array}$$

b. $6 \leq |2x - 4|$



$$\begin{array}{l} 2x-4 \geq 6 \text{ or } 2x-4 \leq -6 \\ 2x \geq 10 \qquad 2x \leq -2 \\ x \geq 5 \qquad \qquad x \leq -1 \end{array}$$

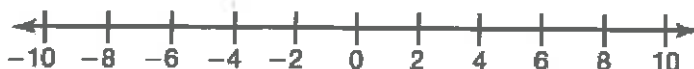
c. $|-5x + 8| + 2 < 25$



$$\begin{array}{l} |-5x+8| < 23 \\ -23 < -5x+8 < 23 \\ -31 < -5x < 15 \\ 31/5 > x > -3 \end{array}$$



d. $|x + 5| > -1$



No solution, -1 is not a distance.

Writing Absolute Value Functions

PROBLEM 2 Too Heavy? Too Light? You're Out!



The official rules of baseball state that all baseballs used during professional games must be within a specified range of weights. The baseball manufacturer sets the target weight of the balls at 145.045 grams on its machines. The specified weight allows for a difference of 3.295 grams. This means the weight can be 3.295 grams greater than or less than the target weight.

1. Write an expression to represent the difference between a manufactured baseball's weight and the target weight. Use w to represent a manufactured baseball's weight.

$$|w - 145.045|$$

2. Write a linear absolute value inequality to represent all baseball weights that are within the specifications.

$$|w - 145.045| \leq 3.295$$

3. Is a baseball that weighs 147.2 grams acceptable? Explain your reasoning.

$$|147.2 - 145.045| = 2.155$$

Yes because 2.155 is less than 3.295.

4. Is a baseball that weighs 141.745 grams acceptable? Explain your reasoning.

$$|141.745 - 145.045| = |-3.3| = 3.3$$

No, because 3.3 is greater than 3.295.

5. Solve the inequality from #2 to determine the greatest and least acceptable baseball weights. Write your answer as an inequality.

$$|w - 145.045| \leq 3.295$$

$$\begin{array}{r} -3.295 \leq w - 145.045 \leq 3.295 \\ +145.045 \qquad +145.045 \end{array}$$

$$141.75 \leq w \leq 148.34$$